

Modelling Subjective Complexity through Cognitive Transitions: Extended Framework

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Abstract—Measuring the operational complexity of arrival traffic is critical for supporting decision-making and enhancing traffic management. In our previous work, we proposed a subjective complexity metric that captures transitions in controllers' workload. However, it has certain limitations, as it does not fully integrate domain-specific knowledge—such as the airspace structure of the terminal maneuvering area—and lacked a systematic approach for defining decision boundaries between complexity levels.

This study addresses these limitations through two key enhancements. First, the airspace structure information is incorporated into trajectory analysis, enabling a better characterization of arrival traffic dynamics. Second, a systematic method is developed for automatically determining decision boundaries between complexity levels, thereby reducing reliance on subjective human judgment.

Using real-world traffic data, the enhanced metric outperforms the baseline in capturing workload variations associated with additional controller-issued manoeuvres. Besides, the automatic boundary determination produces results comparable to manually defined boundaries while improving measurement consistency. Together, these enhancements provide a more reliable assessment of controller workload.

Keywords—Air Traffic Management, Terminal Maneuvering Area, Complexity, Data-driven

I. INTRODUCTION

Complexity measurement, measuring the level of challenge a traffic situation poses to controllers [1], has garnered significant attention over the past two decades, as it constitutes a major component of controller workload [2]–[5] and provides valuable insights for decision-making and sector design in air traffic operations.

Reber [6] introduced the concept of a construct as an internal process that is not directly observable but produces measurable phenomena. Grossberg [7] applied this idea to air traffic control, conceptualising complexity as a construct with both dynamic and static features that influence the rate at which controller workload increases. This view highlights that Air Traffic Control (ATC) complexity is not determined solely by objective sector characteristics but is shaped by the

controller's perception and interaction with the traffic within the sector. Expanding on these foundations, Mogford et al. [8] provided a more detailed view, proposing that complexity arises not only from **source factors** (e.g., air traffic patterns, sector characteristics) but also from **mediating factors** such as equipment quality, individual differences, and the cognitive strategies employed by controllers. Together, these contributions progressively refine the understanding of complexity in air traffic control by integrating structural, perceptual, and cognitive dimensions.

Due to the objective nature of **source factors**, complexity metrics based on them can be leveraged to develop automated air traffic control systems that operate independently of human involvement [9], [10]. Many structural/dynamical metrics fall into this category. Delahaye et al. proposed complexity metrics for both en-route and terminal sectors using nonlinear [11] and linear [12] dynamic system models. These approaches aim to quantify the intrinsic complexity of aircraft trajectories by aligning a vector field with observed positions and velocities. Such metrics are particularly useful in supporting decision-making processes [13]–[15], as they provide the valuable capability to evaluate the degree of traffic organisation independently of traffic density [16].

Due to the subjective nature of **mediating factors**, complexity metrics based on them can inform sector design, as they provide valuable insights into human-centred aspects of controller workload, enabling strategies that better account for cognitive and operational considerations [17]. Nevertheless, previous studies argued that bridging the gap between objective traffic characteristics and perceived complexity requires input from human controllers, since subjective complexity cannot be inferred from traffic data alone [18]. To address this challenge, NASA conducted several studies on dynamic-density, which model complexity as a function of aircraft count and their spatial relationships within a given airspace [19]–[21]. In these studies, human-in-the-loop simulations allowed controllers to manage evolving traffic scenarios and provide



real-time complexity assessments, which were then used to calibrate model coefficients. The dynamic density framework was subsequently extended to incorporate additional complexity factors, reflecting a more comprehensive understanding of perceived complexity in operational contexts [22], [23].

While subjective complexity metrics often rely on controller input during model development, as discussed earlier, this requirement poses practical limitations due to the associated human resource costs. Falling within the human- or subjective-oriented strand of ATC complexity research, our approach draws on Reber’s perspective that a construct—though not directly observable—can be inferred from measurable phenomena [6]. In this context, our previous research [24] focuses on inferring subjective complexity from operational outcomes. Specifically, by analysing the collective dynamics of arrival air traffic operations within the terminal manoeuvring area (TMA), we manually classified these operations into three categories, each corresponding to a distinct level of complexity. While the proposed metric captures shifts in controllers’ perceived complexity, it does not fully incorporate domain-specific knowledge such as the airspace structure of the TMA [25], nor does it offer a systematic methodology for defining decision boundaries between complexity levels, as demonstrated by Lim et al. [26].

To address the two identified research gaps, this study incorporates the influence of standard arrival routes (STARs) into the analysis of radar trajectory data when characterising the collective dynamics of arrival air traffic operations. Furthermore, it introduces a systematic methodology for defining decision boundaries between different levels of operational complexity.

The paper is organized as follows. Section II provides a brief overview of the methodology and findings from our previous work [24], and outlines the existing challenges that this paper aims to address. Section III details the methods employed to handle those identified challenges. Section IV then presents an experimental study, showcasing the application of the framework within a specific TMA. Finally, section V concludes and proposes steps for further research.

II. EXTENSION OF PREVIOUS WORK

Since this study is primarily an extension of our earlier researches [24], [26], this section briefly revisits the core methodology and findings of such a work carried out previously. This allows readers to understand the background and context without needing to refer to the previous paper [24]. We then highlight its key limitations and explain how the current study addresses them.

To visually illustrate the extension from our previous work, Figure 1 presents the overall workflow. The non-coloured components represent the original process proposed in [24], while the coloured steps indicate the key enhancements introduced in this study. These additions address the limitations identified in the earlier work and demonstrate how the current research advances the methodology.

The earlier study first smooths the trajectories using a counter-invariant preprocessing tool, which has been open-sourced on Github [27]. Using the smoothed data, turns are detected by analyzing calibrated curvature [26], [28], which reflects changes in the aircraft’s heading while removing the influence of speed variations. Level-off segments are detected using kernel density estimation (KDE) [29], where peaks in the altitude density indicate periods during which the aircraft maintains nearly constant altitude (Figure 2). Together, these methods provide a coherent framework for segmenting trajectories into turning and level-off phases, enabling detailed analysis of aircraft behavior within the TMA.

With the turning points and level-off segments identified for all trajectories, **without excluding the turning and level-off manoeuvres inherent in STARs**, the previous work directly computed the total number of aircraft in turning and the total number in level-off over time. By plotting them as the x- and y-coordinates, as shown in Figure 3a, the earlier work obtains a moving point in a two-dimensional plane that evolves continuously with time.

As time progresses, the point traces a continuous trajectory, enabling a joint analysis of the dynamic relationship between the number of aircraft in turns and the number of aircraft in level flight. By examining the movement and distribution of this point, as shown in Figure 3b, one can observe not only general patterns and correlations between turning and level-off behaviours over time, but also identify distinct multi-mode distributions. These distributions may reflect variations in air traffic control strategies under different levels of traffic complexity, as certain combinations of turning and level-off activities occur more frequently than others.

Inspired by Reber’s notion of a construct producing measurable phenomena [6], the moving point’s transitions between modes in Figure 3b can be seen as observable manifestations of the underlying traffic dynamics. These manifestations provide a measurable basis for distinguishing different levels of perceived complexity, as variations in the point’s behaviour reflect changes in the workload and coordination demands faced by controllers. Accordingly, in the previous work, decision boundaries associated with the modes were **manually** assigned to separate complexity levels for each region.

Two key limitations exist in our previous work, as briefly revisited above. First, after applying turning and level-off detection, the analysis included all aircraft in turn and in level flight at each timestamp, without excluding standard turns and level-offs that are part of the STAR design, which generally contribute less to controller workload compared with non-standard manoeuvres that often require additional coordination and communication. Second, although the collective dynamics analysis revealed a distinct multi-mode distribution, the boundaries between regions were **manually** defined without a systematic methodology for their determination.

To address these limitations, this study proposes an enhanced methodological framework for learning complexity values. The first enhancement, highlighted in cyan in Figure 1 and detailed in Subsection III-A, incorporates airspace struc-

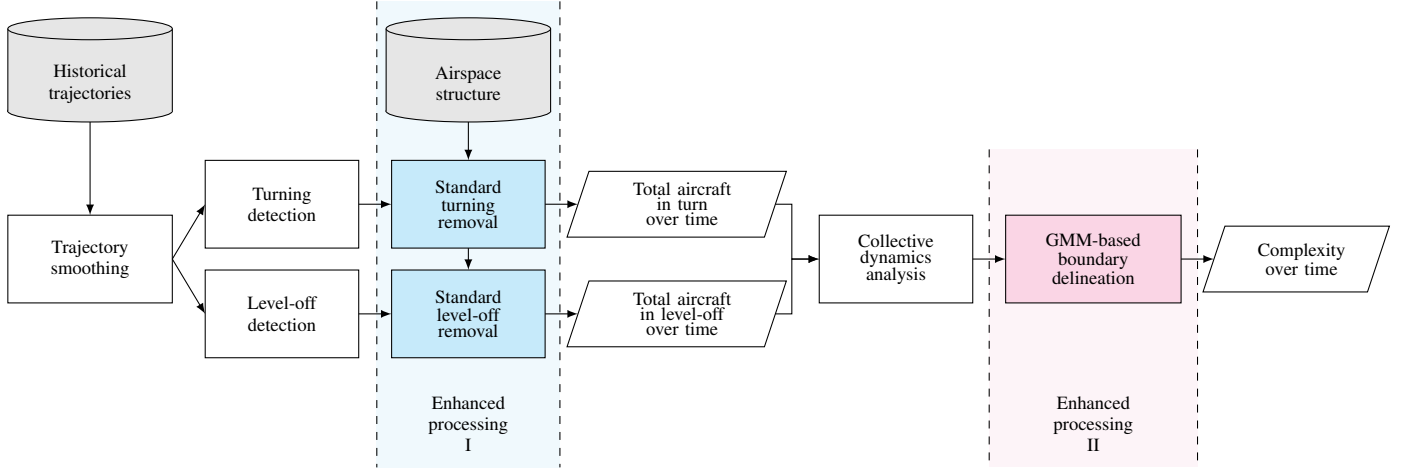


Figure 1. Enhanced methodology framework for learning complexity values. The coloured components indicate the key steps introduced in this study as enhancements to the original workflow.

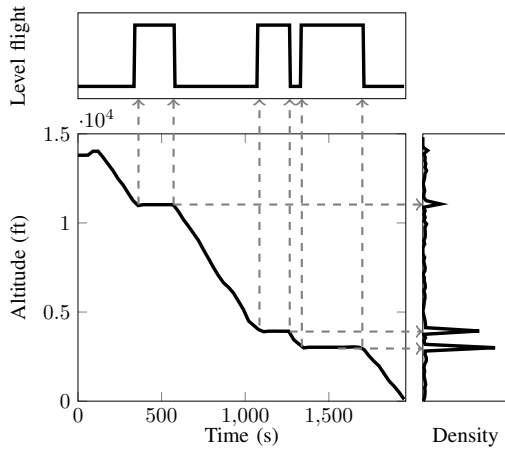


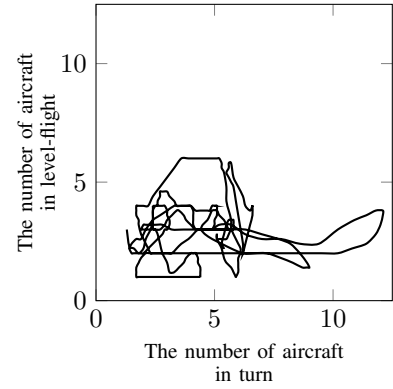
Figure 2. KDE-based level flight detection. If an aircraft maintains level flight at a certain altitude for some time, that altitude exhibits higher density.

ture to identify and remove standard turns and standard level-offs from the detected events. These standard manoeuvres, embedded in STARs, are unlikely to contribute significantly to controller workload. The second enhancement, highlighted in magenta in Figure 1 and described in Subsection III-B, introduces a Gaussian Mixture Model (GMM)-based decision boundary delineation approach to automatically determine decision boundaries in the distribution.

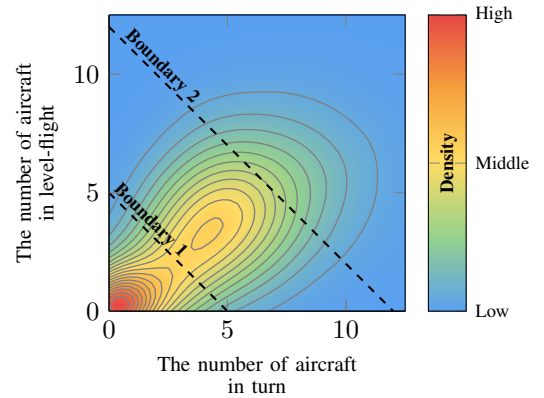
III. METHODOLOGY

A. Enhanced processing I

This subsection aims to integrate domain knowledge to identify and remove standard turns and standard level-offs from the set of detected manoeuvres. As shown in Figure 1, the input comprises detected turns and level-offs obtained from the respective detection processes. The resulting output, as shown in Figure 1, is a time series indicating the number of aircraft performing non-standard turns and the number performing non-standard level-offs.



(a) Moving point trajectory (partial view for clarity)



(b) Moving point density map

Figure 3. Collective dynamics of air traffic derived from curvature- and level-flight analyses. The x-axis represents the number of aircraft in turning (obtained through curvature analysis), and the y-axis represents the number in level flight (obtained through level-flight analysis), both varying over time.

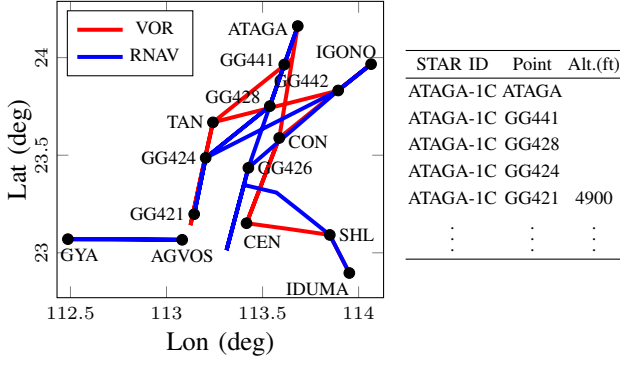


Figure 4. STARs for runways 01/02L/R at Guangzhou Baiyun International Airport (ZGGG, partial view). Each STAR is composed of sequential waypoints, some of which are associated with altitude constraints. These constraints specify the minimum altitude that must be maintained at a given waypoint, resulting in temporary level segments from which level-off information can be inferred.

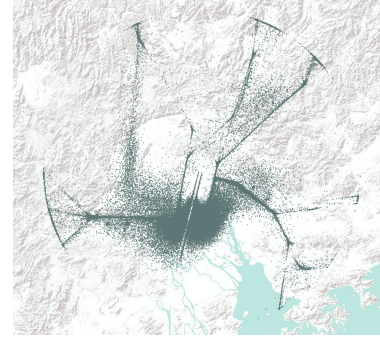
As shown in Figure 4, the airspace structure is utilized to determine whether a turn or level-off follows a standard procedure. It includes detailed information on waypoint locations and altitude constraints associated with standard turns and level-offs. This enables a more accurate assessment of whether each manoeuvring conforms to operational norms.

Figure 5 presents the detected turning points derived from the detection process. In Figure 5a, each dot represents a turning point from historical trajectories. To clarify their spatial distribution, a density estimation is provided in Figure 5b. The results indicate that most turning points are concentrated near published waypoints, with only a small number scattered sparsely along the STAR routes.

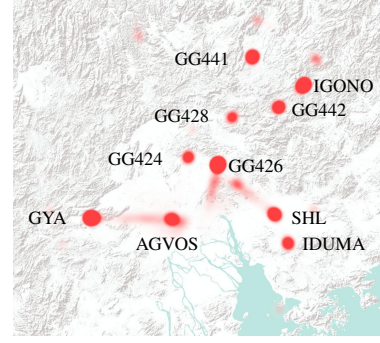
Although waypoint data (Figure 4) and turning point information (Figure 5) are available, they are not sufficient to accurately identify standard turns based solely on the proximity of a turning point to a published waypoint. Some turns occurring near published waypoints are made toward other locations under controller instructions, rather than following the standard route. Therefore, spatial proximity to a waypoint alone does not necessarily indicate a standard turn. To address this gap, a standard turn identification mechanism is developed in Figure 6.

As illustrated in Figure 6a, if a trajectory's two consecutive turns correspond to two consecutive waypoints, those two turns can be treated as standard turns. Otherwise, even if the turns are located at published waypoints, they should not be considered as standard turns. The following are three commonly observed non-standard cases:

- 1) In Figure 6b, the aircraft postpones its first turn, so the turn no longer occurs at the first waypoint. Although the second turn is located at a waypoint, the two turns together form a path stretch and therefore cannot be considered standard turns.
- 2) In Figure 6c, the aircraft makes turns at both waypoints, but they are not consecutive. An additional turn occurs between them, creating a path stretch. Thus, even though



(a) Detected turning points.



(b) Turning point density estimation.

Figure 5. Detected turning points obtained from detection, serving as input for standard turn removal. The size of each red dot is proportional to the number of turning points located at that position.

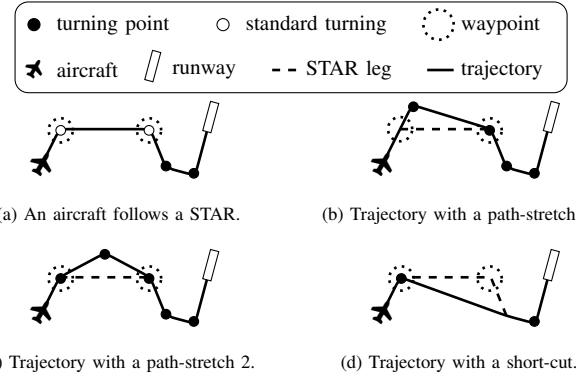


Figure 6. Standard turn identification mechanism. When an arriving aircraft follows a STAR, two consecutive turns in its trajectory should correspond to two consecutive waypoints along the published route. Otherwise, even if the turns are located at the waypoints, they are not considered standard turns.

two turns occur at waypoints, they cannot be regarded as standard turns.

- 3) In Figure 6d, the aircraft may fly directly from the first waypoint to a point beyond the second one, creating a shortcut. Even if the first turn occurs at the waypoint, it is not considered a standard turn.

As suggested in Figure 6d, turns forming shortcut manoeuvres are considered non-standard and contribute to controller workload. While this may seem counter-intuitive, it is logically consistent. Although shortcut turns are more commonly

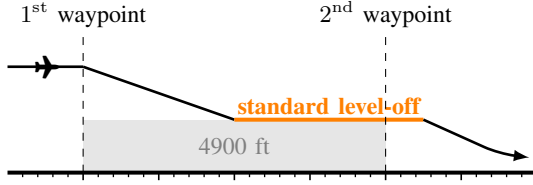


Figure 7. Level flight identification. When an arriving aircraft passes through two consecutive STAR waypoints, with the second waypoint imposing an altitude constraint (e.g., 4900 ft. shown as the shaded area), the corresponding detected level-off segment at that altitude is treated as a standard level-off, even if part of the segment extends beyond the second waypoint.

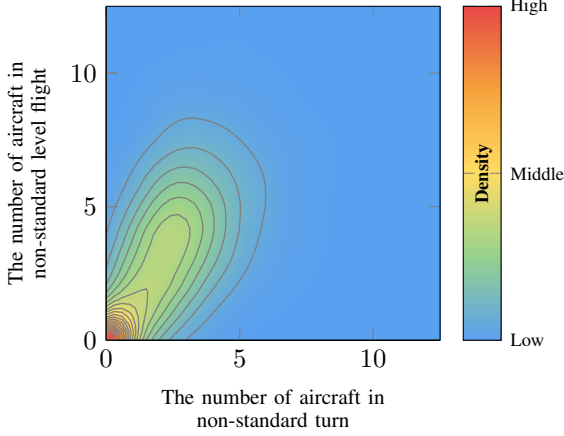


Figure 8. Density map of moving points derived from collective dynamics analysis based on non-standard turns and level-offs.

observed in low-complexity situations, their occurrence is not strictly limited to such scenarios. Shortcuts are fundamentally a traffic sequencing strategy and can be issued in both low- and high-complexity conditions. Compared to standard procedures, instructing an aircraft to perform a shortcut represents an additional controller action and therefore contributes to operational complexity. Moreover, in genuinely low-complexity situations, shortcut turns do not add a large number of additional turns, so the overall number of turns during that period remains small.

The detection of standard level-offs is considerably simpler than that of standard turns. As illustrated in Figure 7, when an arriving aircraft passes through two waypoints—where the second waypoint specifies an altitude constraint (e.g., 4900 ft)—the entire detected level-off segment at that altitude is considered a standard level-off, even if part of the segment extends beyond the second waypoint.

So far the standard turns and standard level-offs are identified and removed. The new moving point density map without considering the standard manoeuvres is shown in Figure 8. Compared with the density map in Figure 3b, the distribution exhibits a contraction toward the lower-left corner, suggesting that a significant number of standard manoeuvres have been successfully excluded.

B. Enhanced processing II

As discussed earlier, the joint analysis of the number of aircraft in turns and in level flight revealed multi-modal

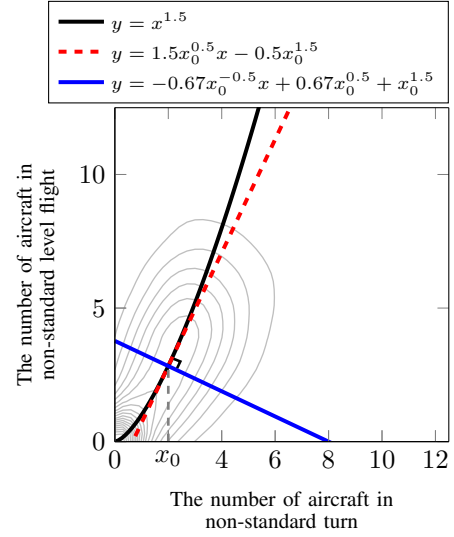


Figure 9. Candidate decision boundary generation.

patterns in their collective behaviour, suggesting that different combinations of these activities may correspond to distinct levels of traffic complexity or control strategies. To objectively capture and separate these patterns, this subsection presents a boundary delineation approach based on a GMM. Specifically, the GMM is employed to decompose the spatial distribution of the moving point into multiple latent components, enabling the identification of potential modes—including those that may be obscured in the overall distribution—and facilitating the systematic definition of decision boundaries.

According to Figure 8, the new distribution of the moving point exhibits a ridge-like structure, this subsection propose a geometric method that leverages the intrinsic directional pattern of the distribution to effectively segment the moving point's distribution. The key idea is to first fit a central curve (referred to as the ridge curve) that captures the primary trend of the distribution [30], [31].

The ridge curve, fitted from the data, is given by the following equation:

$$y = x^{1.5}$$

, and illustrated by the black solid line in Figure 9. At any point $(x_0, x_0^{1.5})$ along this ridge, we first construct the tangent line (red dashed line) to indicate the local direction of the flow, and then the normal line (blue solid line) perpendicular to it. Each normal line serves as a candidate decision boundary, providing a natural axis along which we can evaluate how often the moving point would cross. Because the normals are orthogonal to the main flow, they offer an intuitive and geometrically meaningful way to segment the distribution.

The main objective of decision boundary determination is to identify several effective boundaries that the moving point crosses as few times as possible, in order to ensure the stability of the complexity measurement. Frequent crossings would cause the complexity level to fluctuate constantly, undermining a meaningful assessment. To achieve this, a set of candidate

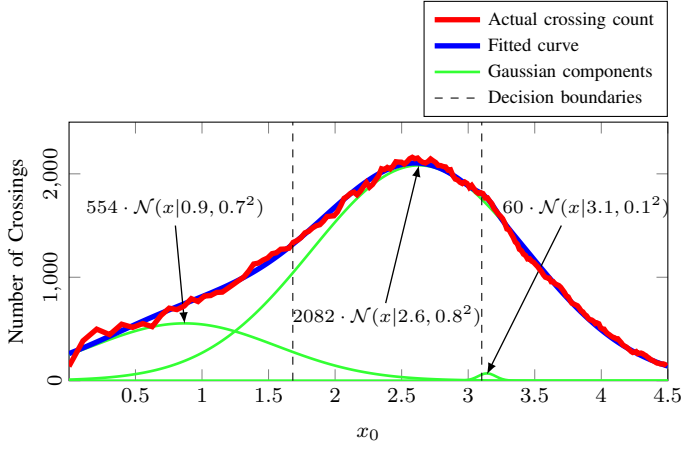


Figure 10. Decision boundary determined based on fitted Gaussian functions. Each decision boundary is determined by Equation 1.

boundaries is generated at different x_0 positions, and each is evaluated by counting the number of crossings. This crossing count serves as a quantitative measure for selecting low-interference, structure-aware segmentation boundaries. The crossing count is shown as the red line in Figure 10.

The crossing count curve generally follows an inverted V-shape, with lower values at both ends and higher values near the centre. In the central region, there is no clear drop in crossing counts, which makes directly selecting the minimum crossing point as a decision boundary meaningless. This phenomenon is likely caused by dominant components in the distribution overshadowing other underlying modes, rendering them less visible in the overall curve.

To address this issue, this study employs GMMs to decompose the distribution into multiple latent components. Each component represents a potential hidden mode, allowing us to systematically identify and define decision boundaries in a way that is not biased by the overshadowing effect of dominant components. In this study, the number of GMM components is set to three. This choice is motivated by the need to enable a direct comparison with previous studies, which also classified complexity into three levels. Additionally, three components are sufficient to provide a good fit to the observed data.

As shown in Figure 10, the blue fitted curve represents the overall GMM approximation, composed of three underlying Gaussian components shown in green. Since these Gaussian components have different spreads, a standard deviation-weighted average is used to estimate the decision boundaries. Specifically, the boundaries are computed using the following expression:

$$x = \frac{\mu_1\sigma_2 + \mu_2\sigma_1}{\sigma_1 + \sigma_2} \quad (1)$$

where μ_1 and μ_2 are the means, and σ_1 and σ_2 are the standard deviations of two consecutive Gaussian components, respectively.

Equation 1 ensures that the boundary is closer to the mean of the component with the smaller standard deviation, as a

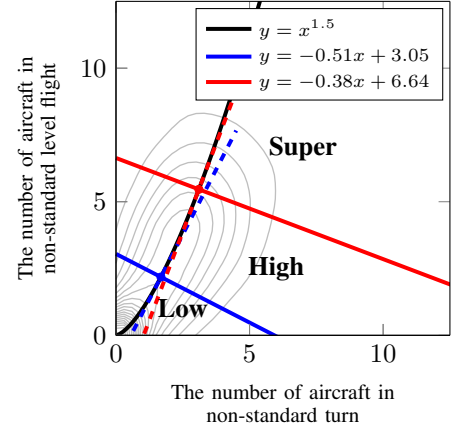


Figure 11. Complexity level determination.

smaller σ indicates a more concentrated distribution. In other words, the boundary is shifted toward the more tightly clustered component, reflecting the intuition that a concentrated component should exert greater influence on the placement of the separation point. By weighting the means according to the other component's standard deviation, the method balances the influence of both components while accounting for their spreads.

Finally, as shown in Figure 11, the two-dimensional space of the moving point is partitioned into three regions using the identified decision boundaries. These regions correspond to three levels of complexity: low, high, and super high. When the moving point enters each region, the corresponding complexity of the arrival air traffic operation at that time is characterized accordingly.

IV. EXPERIMENTAL STUDY

This section presents an experimental study to demonstrate how the proposed metric provides a more effective assessment of traffic complexity compared with its earlier version [24], which is used here as the baseline model. For a fair comparison, the same air traffic operation data source as in the previous study is used. Specifically, arrival trajectories recorded from January 1 to January 7, 2019, comprising 120,961 trajectory points, are used to calibrate the metric parameters, while trajectories from January 30, 2019, within the TMA of Guangzhou Baiyun Airport, are used for validation.

The complexity results of the proposed metric and the baseline metric are shown in Figure 12. Overall, the trends of both metrics exhibit similar profiles. However, the proposed metric consistently assigns complexity levels that are equal to or higher than those of the baseline, reflecting its more aggressive assessment strategy.

To highlight the differences, two representative periods (A and B) were selected for detailed analysis. Period A corresponds to a high complexity level under the proposed metric versus a lower level under the baseline, while period B corresponds to a super-high complexity level under the proposed metric versus a high level under the baseline. These

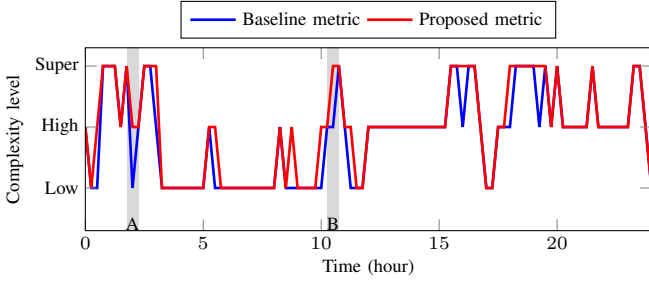


Figure 12. Applying the proposed metric to real air traffic operations. The trends of both metrics exhibit similar profiles. However, the proposed metric (the red line) consistently assigns complexity levels that are equal to or higher than those of the baseline (the blue line).

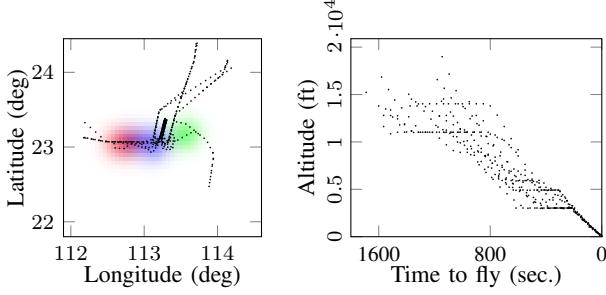


Figure 13. Arrival traffic during period A. The left panel depicts the horizontal trajectories of 34 arriving flights, with colours indicating different manoeuvres: red for holding, blue for path-stretch, and green for short-cut. The right panel shows the corresponding vertical trajectories.

cases effectively illustrate the enhanced sensitivity of the proposed metric in capturing increases in perceived complexity relative to the baseline.

Furthermore, leveraging a trajectory labelling tool [26], this section visualizes additional manoeuvres in the trajectories that reflect controllers' extra workload, particularly radar vectoring that causes aircraft to deviate from STARs. The results are shown in the left panels of Figures 13 and 14, with colours indicating different manoeuvre types: red for holding, blue for path-stretch, and green for short-cut manoeuvres.

In period A (01:45:00 to 02:14:59), the proposed metric indicates a high complexity for the arrival air traffic, whereas the baseline metric does not. Figure 13 is presented to assess which metric better reflects reality. As shown, controllers not only implemented holding, path-stretch, and short-cut manoeuvres but also issued some level-off instructions during this period. Given that holding patterns indicate the runway is operating at or near capacity, requiring controllers to hold some aircraft before landing, the complexity of the arrival traffic should not be considered low. This suggests that the proposed metric provides a more appropriate assessment in such cases.

In Period B (10:15:00 to 10:44:59), the proposed metric indicates super high complexity for the arrival air traffic, whereas the baseline metric does not. Figure 14 is presented to evaluate which metric better reflects reality. Compared with Period A, Period B shows several notable differences: the absence of

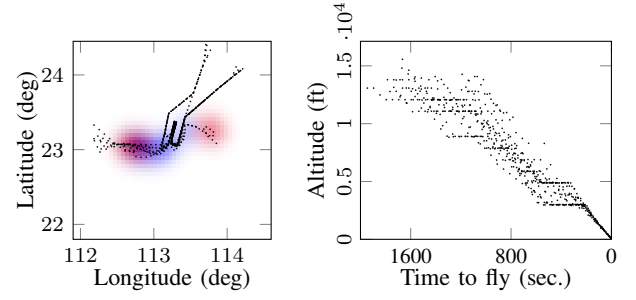


Figure 14. Arrival traffic during period B. The left panel depicts the horizontal trajectories of 34 arriving flights, with colours indicating different manoeuvres: red for holding and blue for path-stretch. The right panel shows the corresponding vertical trajectories.

short-cuts, a greater number of holdings, an increase in level-offs, and spatial overlap between path-stretches and holdings. These features suggest that the traffic situation in Period B was more complex than in Period A, warranting its classification at the super high complexity level. Hence, the proposed metric appears to provide a more appropriate assessment in this case.

To sum up, the experimental study demonstrates that the proposed metric is better suited to capturing actual controller workload, thereby supporting the rationale for removing standard turns and standard level-offs as outlined in subsection III-A. Furthermore, the fact that the metric functions effectively also provides indirect validation of the automated boundary determination method introduced in subsection III-B.

V. CONCLUSION

This study extends our previous work [24] by incorporating the airspace structure information into radar trajectory analysis to better characterise the complexity of arrival air traffic. It further introduces a systematic approach for automatically defining decision boundaries between complexity levels.

Experimental results indicate that removing standard turns and standard level-offs increases the metric's sensitivity to variations in controller workload. The automatically determined decision boundaries yield results broadly comparable to those manually set in our previous study.

The contributions are twofold: improving the consistency of complexity assessment and reducing subjectivity in evaluating controller workload. These refinements offer a more structured basis for analysing controller workload and supporting traffic management decisions.

In future work, two directions will be pursued. First, the proposed metric will be validated through studies involving a small number of controllers or a focus group, to assess how well it reflects perceived workload and operational complexity. Second, the metric will be refined by distinguishing between types of non-standard turns—such as those due to published holdings, unpublished holdings, or vectoring—to provide a more accurate assessment of traffic complexity.

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